The Boston Area Undergraduate Physics Competition

April 21, 2001

Name: ______________________________

School: ______________________________

Year: ______________________________

Address: ______________________________

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e-mail: ______________________________

Phone: ______________________________

Do not turn this page until you are told to do so.

Each of the six problems is worth 10 points.

You have four (4) hours to complete this exam.

Please provide the information requested on this cover sheet. At the end of the exam, hand in this cover sheet with your solutions. You may keep the exam questions.

Show all relevant work in your exam books. Please write neatly. Partial credit will be given for significant progress made toward a correct solution.

You must be enrolled in a full-time undergraduate program to be eligible for prizes.

2001

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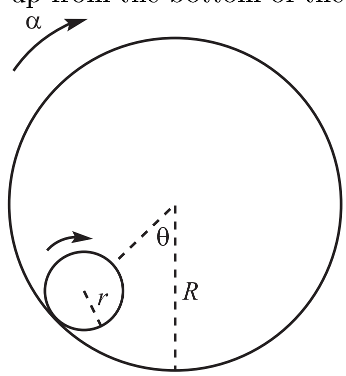
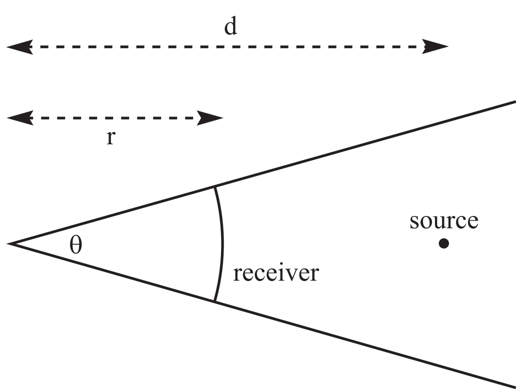
Time: 4 hours

1. A ball (with moment of inertia $I = (2/5)mr^2$) rolls without slipping on the inside of a cylinder of radius R. The cylinder spins around its axis (which points horizontally) with angular acceleration α. What should α be if you wish for the center of the ball to remain motionless at an angle θ up from the bottom of the cylinder (see figure)?

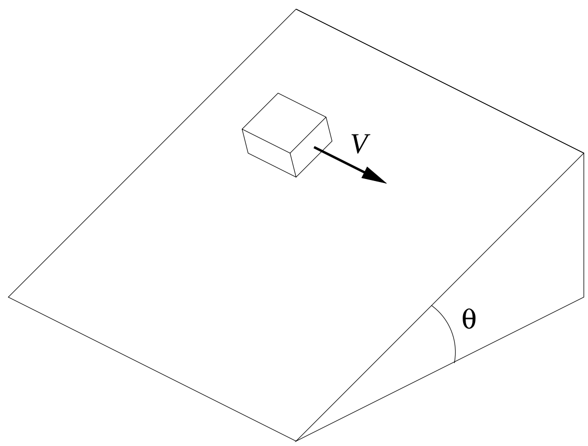


side view

2. A point source emits light (spherically symmetrically) and is located along the axis of a cone (with vertex angle θ), at a distance d from the tip. The inside surface of the cone is reflective. A receiver is located inside the cone. The receiver consists of all points inside the cone that are a distance r from the tip. (See the figure below for a two-dimensional slice of the setup.) What fraction of the light emitted by the source hits the receiver?

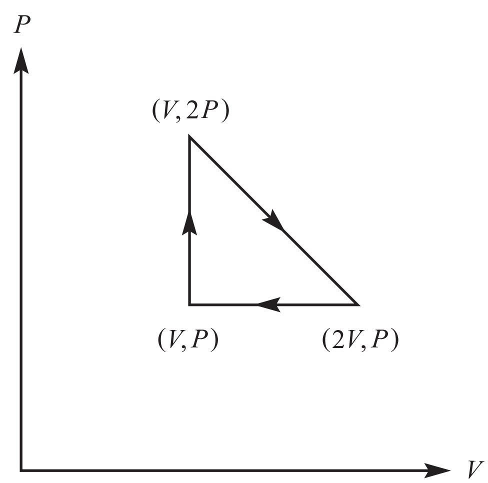


3. A block is placed on a plane inclined at angle θ. The coefficient of friction between the block and plane is $\mu = \tan\theta$. The block is given a kick so that it initially moves with speed V horizontally (i.e., in the direction perpendicular to the direction pointing straight down the plane; see the figure below). What is the speed of the block after a very long time?



4. Find the efficiency of the thermodynamic process shown below. The corners of the triangle are located at the points $(V, 2P)$, (V, P), $(2V, P)$.

   Notes: (1) The "efficiency" of a thermodynamic process is defined to be $\epsilon \equiv (Q_{\text{in}} - Q_{\text{out}})/Q_{\text{in}}$, where Q_{in} and Q_{out} are the quantities of heat added and removed from the system, respectively, (2) the gas is assumed to be an ideal gas, the internal energy of which is $\frac{3}{2}nkT$.



5. A rubber band with initial length L has one end tied to a wall. At $t = 0$, the other end is pulled away from the wall at speed V. (Assume the rubber band stretches uniformly.) At the same time, an ant located at the end not attached to the wall begins to crawl toward the wall, with a speed of u relative to the band. Will the ant reach the wall? If so, how much time will it take?

6. N points in space are connected by a collection of $1\,\Omega$ resistors. The network of resistors is arbitrary, except for the fact that it is "connected" (that is, it is possible to travel between any two points via an unbroken chain of resistors). The number of resistors emanating from any point can be any number from 1 to $N - 1$.

   Consider two points that are connected by a $1\,\Omega$ resistor. The network produces an effective resistance between these two points. What is the sum of the effective resistances across all the $1\,\Omega$ resistors in the network?

   Note: You will receive 2 points for stating the correct answer (which must be supported by a few simple examples), and then 8 points for proving the general result.