The Boston Area Undergraduate Physics Competition

April 22, 1995, 1:00-4:00 pm
Jefferson 250
Harvard University

Each of the six questions is worth 10 points.

1. A cylindrical pipe is positioned with its axis parallel to the ground. The radius is r and the axis is at height h above the ground. What is the minimum initial speed a ball must have in order to be thrown over the pipe? (The ball is thrown from the ground, $h = 0$, and has negligible size.) Consider two cases:

   (a) The ball is allowed to touch the pipe (you may assume the pipe is frictionless).

   (b) The ball is not allowed to touch the pipe.

2. These three sub-questions require only short, one or two sentence, answers.

   (a) A container is divided into two equal parts by a partition. One part contains an ideal gas at temperature T, the other part is a vacuum. The partition is quickly removed. What is the new equilibrium temperature of the gas in the container?

   (b) A semi-infinite wire, carrying a current I, ends on a infinite conducting plane that is perpendicular to the wire. Find the magnetic field at a distance r from the wire and a distance d from the plane.

   (c) A hockey puck, sliding on a frictionless surface, is attached by a piece of string (lying on the surface) to a vertical pole. The puck is given a tangential velocity, and as the string wraps around the pole, the puck spirals in. Explain what, if anything, is wrong with the following statement: "From conservation of angular momentum, the speed of the puck will become greater as the distance from the pole decreases. Hence the kinetic energy increases, and energy is not conserved."

3. Only a single wire is necessary to make a telegraph connection. Both terminals may have metalic objects buried in the ground, making the earth the other wire. Assume that these objects are spheres of radius r buried very deep in the ground (ignore edge effects due to the finite size of the depth). The distance between the terminals is L. What is the resistance (due to the earth) between the terminals? Assume that the resistivity ρ of the earth is uniform, and that $L \gg r$.

4. The semi-infinite circuit shown below is connected to an oscillating emf of the form $V_0 \cos \omega t$. Each inductor has inductance L, and each capacitor has capacitance C.

(a) What is the current (as a function of time) through the leads A, B?

(b) What is the average power delivered by the emf source?

5. A mountain climber wishes to climb a frictionless conical mountain. He wants to do this by throwing a lasso (a rope with a loop) over the top and climbing up along the rope (assume the mountain climber is of negligible height, so that the rope lies along the mountain). At the bottom of the mountain are two stores, one which sells "cheap" lassos (made of a segment of rope tied to loop of rope of *fixed* length), and the other which sells "deluxe" lassos (made of one piece of rope with a loop of *variable* length; the loop's length may change without any friction of the rope with itself).

   When viewed from the side, this conical mountain has an angle α at its peak. For what angles α can the climber climb up along the mountain if he uses:

   (a) a "cheap" lasso and loops it once around the top of the mountain?

   (b) a "deluxe" lasso and loops it once around the top of the mountain?

   (c) a "cheap" lasso and loops it N times around the top of the mountain? (Assume no friction of the rope with itself.)

   (d) a "deluxe" lasso and loops it N times around the top of the mountain? (Assume no friction of the rope with itself.)

6. This problem deals with the terminal velocity of a pencil rolling down an inclined plane. To avoid cumbersome calculations of moments of inertia, in this problem we will approximate the pencil as having all its mass M located on the center axis. To avoid other complications, we will assume that the cross section of the pencil looks like a wheel with six equally spaced spokes and no rim. The lengths of these spokes are all r. (An uncomfortable pencil to handle, indeed; but a much more comfortable one to theorize about.)

   The angle of inclination of the plane is α. Assume there is infinite friction (no slipping) between the pencil and the plane, and that the pencil does not bounce when the end of a "spoke" hits the plane. Assume the plane is very hard so that the "spokes" press into the plane a negligible amount.

   (a) Explain why the speed of the pencil is bounded from above (assuming that it remains in contact with the plane at all times), i.e., why it reaches some (average) terminal velocity.

   (b) Assume that conditions are set up such that the pencil will eventually reach some non-zero terminal (average) velocity (while remaining in contact with the plane at all times). Describe this terminal velocity; you may do this by simply stating the maximum speed of the axis of the pencil in this "steady" state.

(c) What is the minimum angle of inclination α which allows a non-zero terminal velocity to exist? (An initial kick to the pencil is allowed.)

(d) What is the maximum angle of inclination α if the pencil is to stay in contact with the plane at all times?

(e) Do parts (b), (c), and (d) for a pencil with N equally spaced "spokes", where N is very large. In addition to assuming N is very large, you may assume the angle of inclination α is small, and you may use small angle approximations ($\sin\theta \approx \theta$, etc.) where appropriate.

(f) For very large N, what is the maximum possible terminal velocity if the pencil is to remain in contact with the plane at all times?