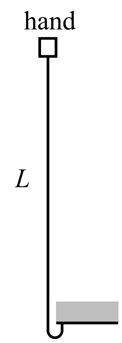
2003
Boston Area Undergraduate Physics Competition

April 26, 2003
Time: 4 hours

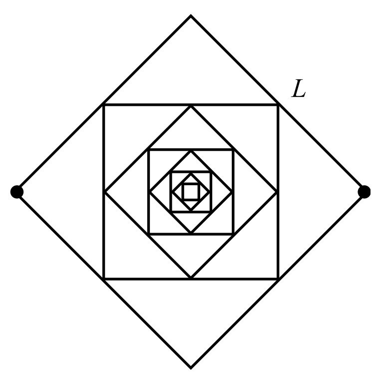
Each of the six questions is worth 10 points.

1. A rope with mass M and length L is held in the position shown below, with one end attached to a support. (Assume that only a negligible length of the rope starts out below the support.) The rope is released. Find the force that the support applies to the rope, as a function of time.



2. A mass is connected to one end of a massless string, the other end of which is connected to a very thin frictionless vertical pole. The string is initially wound completely around the pole (in a very large number of little horizontal circles), with the mass touching the pole. The mass is released, and the string gradually unwinds. What angle does the string make with the pole at the moment it becomes completely unwound?

3. Consider building the following resistor circuit. Start with a square of side length L. Connect the centers of each side to form another square. Connect the centers of each side of this square to form yet another square, and so on, to infinity. What is the resistance between two opposite corners of the original square? (Assume that all the wires in the circuit have the same cross-section and resistivity.) Give your answer in terms of the resistance, R, of a length L of the wire.



4. A charged particle travels into a region in which there is a friction force proportional to the particle's velocity. It travels 10 cm from the point of entry into the region before coming to rest. If a magnetic field of unknown strength is turned on in the region, the particle instead moves along a spiral and comes to rest 6 cm from the point of entry. (This 6 cm is the magnitude of the net displacement, that is, the straight-line distance.) How far from the point of entry will the particle come to rest if the magnetic field is doubled?

5. Consider a ball (with moment of inertia $I = (2/5)MR^2$) which bounces elastically off a surface. Assume that the ball's speed in the direction perpendicular to the surface is the same before and after a bounce. Also, assume that the ball is made of a type of rubber which allows it to not slip on the surface (which has friction) during the bounce. (This implies that the angular and linear motions may affect each other.)

   The ball is projected from the surface of a plane which is inclined at angle θ. The initial velocity of the ball is perpendicular to the plane and has magnitude V. The initial angular velocity is zero. Find the component of the ball's velocity along the plane, immediately after the nth bounce.

6. A ball (with moment of inertia $I = (2/5)MR^2$) rolls without slipping on the inside surface of a fixed cone, whose tip points downward. The half-angle at the vertex of the cone is θ. Initial conditions have been set up so that the ball travels around the cone in a horizontal circle of radius ℓ, with the contact points (the points on the ball that touch the cone) tracing out a given circle (not necessarily a great circle) on the ball.

   What should the radius of the circle of these contact points be, if you want the sphere to travel around the cone as fast as possible? (You may work in the approximation where R is much less than ℓ. Also, assume that the coefficient of friction between the ball and the cone is arbitrarily large.)